Effects of stochastic drifts and time variation on particle diffusion in magnetic turbulence

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The effect on the guiding center trajectories of the stochastic drifts due to the curvature of the stochastic magnetic lines is studied in the first part of this paper. It is shown that the subdiffusive $\sqrt{t}$ behavior of the mean square displacement of the particles cannot exist in a realistic magnetic configuration. The particles undergo a diffusive process even in the absence of the perpendicular collisional diffusion. The anomalous diffusion coefficient is estimated. The second part of this work deals with time-dependent stochastic perturbations of the confining magnetic field. It is shown that the time variation has a strong effect of decorrelating the particles from the magnetic lines. The diffusion coefficient is determined as a function of the correlation time of the stochastic field. The influence of a cross field collisional diffusion on the results of the two problems presented here is also estimated. [S1063-651X(96)10405-0]

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I. INTRODUCTION

A large number of theoretical and experimental studies [1–16] led to the conclusion that the fluctuations of the magnetic field observed in fusion plasmas (tokamak, stellarator, etc.) provide a major contribution to the enhanced particle and energy transport. Even a very small stochastic component of the magnetic field in the radial direction combined with the high velocity motion of the particles along field lines determines high radial displacements. However, these two processes do not produce a radial diffusion but rather a slower time growth of the dispersion of the trajectories (proportional to $\sqrt{t}$). It was shown [3,5,9,15] that only when a supplementary mechanism acts to decouple the particles from the magnetic lines is this subdiffusive behavior dominated asymptotically by a diffusion process. Such a decoupling mechanism, provided by the collisions producing a small cross field diffusion that is strongly enhanced due to the fast parallel motion along the stochastic magnetic lines, is presented in detail in Ref. [5] (see also Ref. [9]).

In the present paper we study two other mechanisms of decoupling the particles from the magnetic lines: these are not related to collisions.

First, we show that there exists an intrinsic decorrelation mechanism in any space-dependent stochastic magnetic field. It consists in the stochastic drifts determined by the gradients of the magnetic field, which are always present along a stochastic magnetic line. As a consequence, the subdiffusive $\sqrt{t}$ behavior of the mean square displacement cannot exist even when the perpendicular collisional diffusion can be neglected. We evaluate the diffusion coefficient determined by the stochastic drifts and show that, in weakly collisional plasmas, this mechanism may provide the major contribution to trajectory dispersion. This problem was previously studied by Coronado, Vitela, and Akcasu [6] starting from a model similar to ours and by Mynick and Krommes in Ref. [10] for the magnetic configuration of the tokamak in a more general context including stochasticity criteria. However, both studies consider only a constant velocity motion of the particles along perturbed magnetic lines. In our model the parallel velocity is a stochastic function of time determined by collisions. We show that, even in the weakly collisional limit, the results of the two models are different and that the constant velocity is not a good approximation of the physical problem. This is in agreement with the conclusion of several papers [4,8,14] that the fluctuating nature of the particle velocity along field lines is essential for an appropriate description of plasma transport processes in magnetic turbulence.

The decorrelation of the particles from the magnetic lines can also be produced by the time variation of the stochastic magnetic field. We show that this mechanism is very efficient when the correlation time of the fluctuating field τ_c is of the order of the inverse of the collision frequency ν and that particle diffusion is strongly enhanced in these conditions. We determine the diffusion coefficient as a function of τ_c and demonstrate that the collisional cross field diffusivity has no decisive influence on the shape of this curve. This problem was previously treated in several papers [6,9,11,12]. Our results are in qualitative agreement with the heuristic analysis presented in Ref. [9] and with part of the conclusions obtained in Ref. [11] from numerical calculations of the confinement time in the case of a single coherent perturbation of the magnetic field. In Ref. [6], the problem is treated in a simplified frame (stochastic magnetic field depending only on time or space-time fluctuating field but with constant velocity parallel motion) that prevents a direct comparison with our results. In Ref. [12], the time dependence of

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the stochastic magnetic field is folded into the Lagrangian correlation function, which is introduced arbitrarily and results that are typical for the static case are eventually obtained.

Our approach is based on Langevin-type equations that describe both magnetic lines and particle guiding center trajectories. We use the methods and some of the results obtained in our previous paper [5], which contains a study of the stochastic magnetic line configuration and of particle motion in such fields. The Lagrangian nonlinearity determined by the dependence of the fluctuating field on spatial position appears there to play an essential role in particle-field line separation due to perpendicular collisions. The influence of the stochastic drift on the particle mean square displacement is shown to act through the Lagrangian nonlinearity as well but the diffusion induced by the time variation of the stochastic magnetic field is not conditioned by the chaoticity of the latter.

The present work belongs to a series of papers (initiated by [4], [5], and [13]) which is devoted to various aspects of the problem of particle and energy transport in regions where the magnetic field is stochastic. The text is organized as follows. The model and the system of equations describing the particle guiding center trajectories are presented in Sec. II. It also contains the statistical assumptions about the fluctuating quantities: magnetic field and collisional velocity. The model is rather general. It contains three decorrelation mechanisms: the perpendicular collisional velocity, the stochastic curvature drifts, and the time dependence of the stochastic magnetic field. Our strategy in dealing with this problem is to study first the effect of each of the three decoupling mechanisms taken separately. The decorrelation due to collisions was presented in our previous paper [5] while the intrinsic mechanism of particle-field line decoupling is studied in Sec. III of the present paper and the time-dependence-induced diffusion in Sec. V. Then the combined action of the decorrelation mechanisms is analyzed: stochastic drifts and collisions in Sec. IV and collisional particles in time dependent stochastic magnetic field in Sec. VI. The results obtained there allow us to draw conclusions about the full physical problem, in the frame of the Langevin model presented in Sec. II. They are summarized in Sec. VII.

II. THE MODEL

We consider a shearless slab geometry for the confining magnetic field with a strong (constant) component B_0 along the z axis and a fluctuating perpendicular component $B_0\mathbf{b}(\mathbf{x})$ in the (x,y) plane:

$$\mathbf{B}(\mathbf{x},t)=B_0(\mathbf{e}_z+b_x(\mathbf{x},t)\mathbf{e}_x+b_y(\mathbf{x},t)\mathbf{e}_y). \tag{1}$$

The time variation is slow (with characteristic frequencies $\omega \lesssim 1$ MHz, as observed in tokamak plasmas [16]) so that the induced electric field is negligible. The stochastic component $\mathbf{b}$ is represented by a vector potential $\Psi(\mathbf{x},t)\mathbf{e}_z$ in order to have the condition of zero divergence automatically fulfilled:

$$\mathbf{b}(\mathbf{x},t)=\boldsymbol{\nabla}\times(\Psi(\mathbf{x},t)\mathbf{e}_z). \tag{2}$$

The configuration of the magnetic lines is studied in detail in our previous paper [5]. The particle guiding centers move along the field lines with a velocity that can be modeled by a stochastic function of time. We assume that the unperturbed field is strong enough so that the motion of the particles can be described in the drift approximation. In Ref. [5] the perpendicular drift motion is neglected and the equations of motion are obtained by combining the field line equation with the collisional velocity parallel and perpendicular to the magnetic field, respectively. We are now interested in evaluating the influence of the drifts on particle behavior and also in studying the effect of time variation of the stochastic magnetic field. Thus, the guiding center trajectories $\mathbf{x}_p(t)$ are described by the following set of equations:

$$\frac{d}{dt}x_p(t)=b_x[\mathbf{x}_p(t),t]\,\eta_\parallel(t)+v_{Dx}[\mathbf{x}_p(t),t]+\eta_\perp^x(t), \tag{3}$$

$$\frac{d}{dt}y_p(t)=b_y[\mathbf{x}_p(t),t]\,\eta_\parallel(t)+v_{Dy}[\mathbf{x}_p(t),t]+\eta_\perp^y(t), \tag{4}$$

$$\frac{d}{dt}z_p(t)=\eta_\parallel(t). \tag{5}$$

The drift velocity $\mathbf{v}_D$ is given by [17]

$$\mathbf{v}_D=\frac{1}{\Omega}\,\mathbf{n}\times[\mu\boldsymbol{\nabla}B+v_\parallel^2(\mathbf{n}\cdot\boldsymbol{\nabla})\mathbf{n}]+\frac{v_\parallel}{\Omega}\left(\mathbf{n}\times\frac{\partial\mathbf{n}}{\partial t}\right), \tag{6}$$

where $\Omega=eB/mc$ is the gyration frequency, $\mathbf{n}=\mathbf{B}/B$ is the unit vector along the field line, $\mu=v_\perp^2/2B$ is the magnetic moment, and $v_\parallel,v_\perp$ are the components of the particle velocity parallel and perpendicular to the reference magnetic field, respectively. The z component of the drift velocity is neglected since it combines with the much larger collisional parallel velocity. We note that the Lagrangian nonlinearity consisting in the trajectory dependence appears in the first two terms in the right-hand side (rhs) of Eqs. (3) and (4).

The drift determined by the time variation of the magnetic field [third term in Eq. (6)] will be neglected since it is much smaller than the first two terms [see Ref. [17], Chap. 1].

Two additional approximations can be introduced in Eqs. (3) and (4) although they are not compulsory in developing the model. They simplify considerably the calculations without sensibly affecting the results. First, since the amplitude of the magnetic field fluctuations is very small, we can neglect the terms in b^n, $n\geqslant 2$, in the drift velocity and retain only the dominant, first order, terms:

$$v_{Dx}\cong-\frac{v_\parallel^2}{\Omega}\frac{\partial b_y}{\partial z},\quad v_{Dy}\cong\frac{v_\parallel^2}{\Omega}\frac{\partial b_x}{\partial z}. \tag{7}$$

Secondly, the parallel velocity $v_\parallel$ appearing in Eq. (7) is the particle collisional velocity along magnetic lines [$v_\parallel\equiv\eta_\parallel(t)$]. Thus, the drift velocity is a doubly stochastic process: it fluctuates due to $\mathbf{b}$ and due to collisions. An important simplification is obtained if we average the drift over the fluctuating parallel velocity. This is an acceptable approximation since the drift velocity does not depend on the sign of the parallel velocity. We have also checked that in both cases there is no correlation between the two terms in the rhs of Eqs. (3) and (4). Thus, in doing this approximation, we do not lose any important cross term correlation effect. With these simplifications, the system (3)–(5) becomes

$$\frac{d}{dt} x_p(x) = b_x(\mathbf{x}_p(t),t)\eta_\parallel(t) - \frac{V_T^2}{\Omega} b_{y,z}(\mathbf{x}_p(t),t) + \eta_\perp^x(t), \tag{8}$$

$$\frac{d}{dt} y_p(t) = b_y(\mathbf{x}_p(t),t)\eta_\parallel(t) + \frac{V_T^2}{\Omega} b_{x,z}(\mathbf{x}_p(t),t) + \eta_\perp^y(t), \tag{9}$$

$$\frac{d}{dt} z_p(t) = \eta_\parallel(t), \tag{10}$$

where $b_{n,z} = \partial b_n/\partial z$, $n = x,y$.

Equations (8)–(10) must be completed by specifying the statistical properties of the random quantities. We assume (as in Ref. [5]) that both parallel and perpendicular collisional velocities $\eta_\parallel$ and $\eta_\perp$, respectively, have zero average and are modeled by a Gaussian colored noise:

$$\langle \eta_\parallel(t)\eta_\parallel(t')\rangle = \chi_\parallel \nu \exp(-\nu|t-t'|) \equiv R(|t-t'|), \tag{11}$$

$$\langle \eta_\perp^x(t)\eta_\perp^x(t')\rangle = \langle \eta_\perp^y(t)\eta_\perp^y(t')\rangle = \chi_\perp \nu \exp(-\nu|t-t'|) \equiv R_\perp(|t-t'|), \tag{12}$$

$$\langle \eta_\parallel(t)\eta_\perp^x(t')\rangle = \langle \eta_\parallel(t)\eta_\perp^y(t')\rangle = \langle \eta_\perp^x(t)\eta_\perp^y(t')\rangle = 0, \tag{13}$$

where ν is interpreted as the collision frequency of the plasma, $\chi_\parallel$ as the (classical) parallel diffusion coefficient, and $\chi_\perp$ as the cross field diffusion coefficient. In terms of the thermal velocity $V_T = \sqrt{2T/m}$ (m is the mass of the particle and T the temperature of the plasma), $\chi_\parallel = V_T^2/2\nu$ and $\chi_\perp = V_T^2\nu/2\Omega^2$.

For the description of the fluctuating magnetic field, we make an assumption about the statistical properties of the vector potential $\Psi(\mathbf{x},t)\mathbf{e}_z$ and then derive from it the statistical characteristics of the magnetic field and of its gradients (as in Ref. [5]). This procedure ensures the zero divergence condition for the fluctuating magnetic field. $\Psi(\mathbf{x},t)$ is taken as a Gaussian random field, spatially homogeneous, isotropic in the (x,y) plane, and stationary. The Eulerian autocorrelation function is taken as

$$\mathcal{A}(\mathbf{r},\tau) \equiv \langle \Psi(\mathbf{x}+\mathbf{r},t+\tau)\Psi(\mathbf{x},t)\rangle = \beta^2\lambda_\perp^2 \exp\left(-\frac{r_z^2}{2\lambda_\parallel^2} - \frac{r_\perp^2}{2\lambda_\perp^2}\right)\exp\left(-\frac{|\tau|}{\tau_c}\right). \tag{14}$$

Here, two characteristic lengths are defined: the parallel correlation length $\lambda_\parallel$ and the perpendicular correlation length $\lambda_\perp$, and a characteristic time τ_c. The dimensionless parameter β is a measure of the intensity of the fluctuations of the magnetic field.

Using the methods presented in Ref. [5], we derive the Eulerian correlations for the magnetic field components:

$$\mathcal{B}_{mn}(\mathbf{r},\tau) \equiv \langle b_m(\mathbf{x}+\mathbf{r},t+\tau)b_n(\mathbf{x},t)\rangle = \langle b_m(\mathbf{r},\tau)b_n(\mathbf{0},0)\rangle \tag{15}$$

and for their gradients $b_{m,\alpha}(\mathbf{x}) \equiv \partial b_m(\mathbf{x})/\partial\alpha$, $m = x,y$, and $\alpha = x,y,z$:

$$\mathcal{B}_{mn}^{\alpha\beta}(\mathbf{r},\tau) \equiv \langle b_{m,\alpha}(\mathbf{x}+\mathbf{r},t+\tau)b_{n,\beta}(\mathbf{x},t)\rangle = \langle b_{m,\alpha}(\mathbf{r},\tau)b_{n,\beta}(\mathbf{0},0)\rangle. \tag{16}$$

We extend this notation to the Eulerian correlation function of a field component b_m with a gradient $b_{n,\alpha}$ by introducing the value $\alpha = 0$, which means that no derivative is taken on the corresponding component of $\mathbf{b}$. Nonzero expressions are obtained for all the components of the matrix $\mathcal{B}_{mn}$ and of the tensor $\mathcal{B}_{mn}^{\alpha\beta}$. All are proportional to $\mathcal{A}(\mathbf{r},\tau)$, the correlation of the potential Ψ and contain various factors that are polynomials in the components r_x, r_y, r_z of the distance $\mathbf{r}$ between the two points.

Using these expressions, it is easy to show that the fluctuating drift velocity (6) has zero average.

An order of magnitude estimation shows that the drift terms are much smaller than the parallel motion terms (their ratio is of the order $\rho_L/\lambda_\parallel$ where ρ_L is the Larmor radius). However, the small stochastic drift can have a stronger contribution than the collisional perpendicular diffusivity if $\chi_\perp < \chi_\parallel(\rho_L^2/\lambda_\parallel^2)\beta^2$. For the classical transport model, which gives $\chi_\parallel/\chi_\perp = \Omega^2/\nu^2$, this condition becomes $\lambda_{\rm mfp}\beta/\lambda_\parallel > 1$ ($\lambda_{\rm mfp}$ is the parallel mean free path of the particles), hence it corresponds to high temperature, weakly collisional plasmas. An extended and very clear derivation of the equations of particle motion relevant for the transport studies in the electrostatic and magnetic turbulence in tokamak plasmas is presented in Ref. [6]. The system (8)–(10) is in agreement with the model presented there.

III. AN INTRINSIC PROCESS OF PARTICLE-FIELD LINE DECORRELATION

The aim of the present study is to answer the following question: could the stochastic ∇b drifts provide a decorrelation mechanism of the particles from the magnetic lines? If the decorrelation exists, then the subdiffusive behavior of the particle mean square displacement is not possible and the particles diffuse even in the absence of the perpendicular collisional diffusion. In order to answer this question, we neglect in this section the collisional perpendicular stochastic velocities in Eqs. (8) and (9) [$\boldsymbol{\eta}_\perp(t) = \mathbf{0}$] and consider a static stochastic perturbation of the main field [$\mathbf{b} = \mathbf{b}(\mathbf{x})$ and $\tau_c \to \infty$ in the Eulerian correlation function for the potential Ψ, Eq. (14)].

We use the same method as in Ref. [5], which consists of studying the time evolution of the perpendicular deviation $\Delta\mathbf{x}_p(t)$ of a particle from the magnetic line on which it was located at time 0:

$$\Delta\mathbf{x}_p(t) = \mathbf{x}_{p\perp}(t) - \mathbf{x}_{m\perp}[z_p(t)], \tag{17}$$

where $\mathbf{x}_{m\perp}[z_p(t)]$ is the position where the particle would be at time t had it followed the initial field line. By definition $\Delta\mathbf{x}_p(0) = 0$. The evolution equation for this quantity follows from Eqs. (8)–(10):

$$\frac{d}{dt}\Delta\mathbf{x}_p(t) = \{\mathbf{b}[\mathbf{x}_{p\perp}(t),z_p(t)] - \mathbf{b}[\mathbf{x}_{m\perp}(z_p(t)),z_p(t)]\}\frac{dz_p(t)}{dt} + \mathbf{v}_D[\mathbf{x}_{p\perp}(t),z_p(t)]. \tag{18}$$

The statistical description of $\Delta\mathbf{x}_p(t)$ requires the determination of (at least) three moments: $\langle \Delta x_p^2(t)\rangle_b$, $\langle \Delta y_p^2(t)\rangle_b$, and $\langle \Delta x_p(t)\Delta y_p(t)\rangle_b$, where the average is taken over the statistical ensemble of the stochastic magnetic field. We use here the method presented in detail in Ref. [5]. Similar problems were treated previously in Refs. [8] and [9].

Being interested in the first stage of the evolution of the distance particle-field line ($|\Delta x_p^2(t)|,|\Delta y_p^2(t)| \ll \lambda_\perp^2$) we linearize Eq. (18) around the magnetic line $\mathbf{x}_{m\perp}[z_p(t)]$:

$$\frac{d}{dt}\Delta x_p(t) \cong b_{x,x}[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)]\,\eta_\parallel(t)\Delta x_p(t) + b_{x,y}[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)]\,\eta_\parallel(t)\Delta y_p(t) - \frac{V_T^2}{\Omega}\left\{\frac{\partial}{\partial z} b_y[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)] + \frac{\partial}{\partial z} b_{y,x}[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)]\Delta x_p(t) + \frac{\partial}{\partial z} b_{y,y}[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)]\Delta y_p(t)\right\}, \tag{19}$$

$$\frac{d}{dt}\Delta y_p(t) \cong b_{y,x}[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)]\,\eta_\parallel(t)\Delta x_p(t) + b_{y,y}[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)]\,\eta_\parallel(t)\Delta y_p(t) + \frac{V_T^2}{\Omega}\left\{\frac{\partial}{\partial z} b_x[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)] + \frac{\partial}{\partial z} b_{x,x}[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)]\Delta x_p(t) + \frac{\partial}{\partial z} b_{x,y}[\mathbf{x}_{m\perp}[z_p(t)],z_p(t)]\Delta y_p(t)\right\}. \tag{20}$$

We note that in these equations all the magnetic field components and gradients are evaluated in the (x,y) plane along the magnetic line (not along the trajectory). The z component of the particle trajectory $z_p(t)$ is here considered as a given function of time. Thus, we treat the doubly stochastic process in two steps: we first consider the stochastic magnetic field, solve the Lagrangian nonlinearity, and obtain the square relative displacement averaged over $\mathbf{b}$, and then we average this result over the stochastic collisions. We have checked, in the simpler case where the stochastic drifts are neglected, that the order of performing the averages over $\mathbf{b}$ and $\eta_\parallel$ does not influence the result: the two operations commute.

Multiplying Eqs. (19) and (20) by $\Delta x_p(t),\Delta y_p(t)$, ensemble averaging over the realizations of the stochastic magnetic field, then treating the resulting equations in the spirit of the quasilinear approximation and using the well-known asymptotic Markovianization procedure [17], one obtains a rather complicated system of coupled equations for the moments (see Ref. [5] for more details on this long but straightforward calculation). The coefficients appearing in this system are time integrals of the Lagrangian correlations of various gradients and components of the magnetic field. The latter represent the Lagrangian version of the Eulerian correlations defined in Eqs. (15) and (16). Using as in Ref. [5] the Corrsin approximation [18,19], the Lagrangian correlations can be related to the corresponding Eulerian correlations by

$$\mathcal{L}_{mn}^{\alpha\beta}(\zeta) = \int d\mathbf{r}_\perp \mathcal{B}_{mn}^{\alpha\beta}(\mathbf{r}_\perp,\zeta)\,\gamma(\mathbf{r}_\perp,\zeta), \tag{21}$$

where $\gamma(\mathbf{r}_\perp,\zeta)$ is the probability of finding the current point on a field line at the perpendicular position $\mathbf{r}_\perp$ when a parallel distance ζ is covered. Its definition and estimation are given in Ref. [5]. We recall that $z_p(t)$ is, at this stage, a function of time and thus ζ denotes the parallel position at a given time moment. Using this equation and the expressions of the Eulerian correlations, one finds that, for the *homogeneous*, *gyrotropic*, and *stationary* turbulence we are studying, the table of correlations simplifies considerably in the Lagrangian frame. Some of them are zero and all the others can be expressed in terms of two scalar functions $\mathcal{L}(\zeta)$ and $\mathcal{K}(\zeta)$ (see Ref. [5]):

$$\mathcal{L}_{mn}^{00}(\zeta) = \delta_{mn}\mathcal{L}(\zeta), \tag{22}$$

$$\mathcal{L}_{xx}^{xx}(\zeta) = \mathcal{L}_{yy}^{yy}(\zeta) = -\mathcal{L}_{xy}^{xy}(\zeta) = -\mathcal{L}_{xy}^{yx}(\zeta) = \mathcal{K}(\zeta), \tag{23}$$

$$\mathcal{L}_{xx}^{yy}(\zeta) = \mathcal{L}_{yy}^{xx}(\zeta) = 3\mathcal{K}(\zeta), \tag{24}$$

$$\mathcal{L}_{xx}^{xy}(\zeta) = \mathcal{L}_{xy}^{yy}(\zeta) = \mathcal{L}_{yy}^{xy}(\zeta) = \mathcal{L}_{yx}^{xx}(\zeta) = 0, \tag{25}$$

$$\mathcal{L}_{mn}^{zz}(\zeta) = \frac{1}{\lambda_\parallel^2}\left(1 - \frac{\zeta^2}{\lambda_\parallel^2}\right)\delta_{mn}\mathcal{L}(\zeta), \tag{26}$$

$$\mathcal{L}_{mn}^{lz}(\zeta) = \mathcal{L}_{mn}^{l0}(\zeta) = 0, \quad l = x,y, \tag{27}$$

$$\mathcal{L}_{mn}^{z0}(\zeta) = -\frac{\zeta}{\lambda_\parallel^2}\,\delta_{mn}\mathcal{L}(\zeta), \tag{28}$$

where $\mathcal{L}(\zeta)$ is the solution of the integral equation deduced in [5]:

$$\mathcal{L}(\zeta) = \beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right)\frac{\lambda_\perp^4}{[\lambda_\perp^2 + 2\int_0^\zeta d\zeta_1(\zeta-\zeta_1)\mathcal{L}(\zeta_1)]^2} \tag{29}$$

and $\mathcal{K}(\zeta)$ is the following function of $\mathcal{L}(\zeta)$:

$$\mathcal{K}(\zeta) = \beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right)\frac{\lambda_\perp^4}{[\lambda_\perp^2 + 2\int_0^\zeta d\zeta_1(\zeta-\zeta_1)\mathcal{L}(\zeta_1)]^3}. \tag{30}$$

The equations for the moments become

$$\frac{d}{dt}\langle \Delta x_p^2(t)\rangle_b = 2\int_0^t d\tau\, \mathscr{K}(\xi)\,\eta_\parallel(\tau)\,\eta_\parallel(t)\langle \Delta x_p^2(t)\rangle_b + 6\int_0^t d\tau\, \mathscr{K}(\xi)\,\eta_\parallel(\tau)\,\eta_\parallel(t)\langle \Delta y_p^2(t)\rangle_b - 4\,\frac{V_T^2}{\Omega}\int_0^t d\tau\,\frac{\xi}{\lambda_\parallel^2}\,\mathscr{K}(\xi)\,\eta_\parallel(\tau)$$
$$\times\langle \Delta x_p(t)\Delta y_p(t)\rangle_b - 12\,\frac{V_T^2}{\Omega}\int_0^t d\tau\,\frac{\xi}{\lambda_\parallel^2}\,\mathscr{K}(\xi)\,\eta_\parallel(t)\langle \Delta x_p(t)\Delta y_p(t)\rangle_b + 14\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{K}(\xi)$$
$$\times\langle \Delta x_p^2(t)\rangle_b + 2\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{K}(\xi)\langle \Delta y_p^2(t)\rangle_b + 2\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{L}(\xi), \tag{31}$$

$$\frac{d}{dt}\langle \Delta y_p^2(t)\rangle_b = 6\int_0^t d\tau\, \mathscr{K}(\xi)\,\eta_\parallel(\tau)\,\eta_\parallel(t)\langle \Delta x_p^2(t)\rangle_b + 2\int_0^t d\tau\, \mathscr{K}(\xi)\,\eta_\parallel(\tau)\,\eta_\parallel(t)\langle \Delta y_p^2(t)\rangle_b + 4\,\frac{V_T^2}{\Omega}\int_0^t d\tau\,\frac{\xi}{\lambda_\parallel^2}\,\mathscr{K}(\xi)\,\eta_\parallel(\tau)$$
$$\times\langle \Delta x_p(t)\Delta y_p(t)\rangle_b + 12\,\frac{V_T^2}{\Omega}\int_0^t d\tau\,\frac{\xi}{\lambda_\parallel^2}\,\mathscr{K}(\xi)\,\eta_\parallel(t)\langle \Delta x_p(t)\Delta y_p(t)\rangle_b + 2\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{K}(\xi)\langle \Delta x_p^2(t)\rangle_b$$
$$+14\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{K}(\xi)\langle \Delta y_p^2(t)\rangle_b + 2\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{L}(\xi), \tag{32}$$

$$\frac{d}{dt}\langle \Delta x_p(t)\Delta y_p(t)\rangle_b = -4\int_0^t dt\, \mathscr{K}(\xi)\,\eta_\parallel(\tau)\,\eta_\parallel(t)\langle \Delta x_p(t)\Delta y_p(t)\rangle_b + 12\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{K}(\xi)\langle \Delta x_p(t)\Delta y_p(t)\rangle_b$$
$$+2\,\frac{V_T^2}{\Omega\lambda_\parallel}\int_0^t d\tau\,\xi\mathscr{K}(\xi)[\eta_\parallel(\tau)+3\,\eta_\parallel(t)][\langle \Delta x_p^2(t)\rangle_b - \langle \Delta y_p^2(t)\rangle_b], \tag{33}$$

where $\xi \equiv z_p(t) - z_p(\tau)$.

We first note that the equation for $\langle \Delta x_p \Delta y_p\rangle_b$ is homogeneous (it does not contain a source term as the other two). Thus, the solution corresponding to the initial condition

$$\Delta x_p(0) = \Delta y_p(0) = 0 \tag{34}$$

is $\langle \Delta x_p(t)\Delta y_p(t)\rangle_b = 0$. This means that the stochastic drifts do not generate a cross correlation in particle trajectories. The first two terms in the rhs of Eqs. (31) and (32) are the same as those appearing in the equation for the distance between two magnetic lines [Eqs. (54) in Ref. [5]] and the last five are produced by the stochastic drifts. Particularly important is the last term, which is a source term. The solution of (31) and (32) corresponding to the initial condition (34) is obtained from the simpler equation, which describes the evolution of $\langle \Delta r_p^2(t)\rangle_b = \langle \Delta x_p^2(t)\rangle_b + \langle \Delta y_p^2(t)\rangle_b$:

$$\frac{d}{dt}\langle \Delta r_p^2(t)\rangle_b = \left[8\int_0^t d\tau\, \mathscr{K}(\xi)\,\eta_\parallel(\tau)\,\eta_\parallel(t) + 16\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{K}(\xi)\right]\langle \Delta r_p^2(t)\rangle_b + 4\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t d\tau\left(1-\frac{\xi^2}{\lambda_\parallel^2}\right)\mathscr{L}(\xi). \tag{35}$$

Thus, the stochastic drifts determine a nonhomogeneous (decoupling) term and a modification of the exponentiation rate [second term in the square bracket in Eq. (35)]. However, the latter is very small compared with the first term in the square bracket and will be neglected for simplicity. The solution of Eq. (35) with the initial condition (34) is

$$\langle \Delta r_p^2(t)\rangle_b = 4\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t dt_1 f(t_1) \times \exp\left(8\int_{z_p(t_1)}^{z_p(t)} d\zeta \int_0^\zeta d\zeta_1\, \mathscr{K}(\zeta_1-\zeta)\right), \tag{36}$$

where

$$f(t_1) = \int_0^{t_1} d\tau\left(1-\frac{(z_p(t_1)-z_p(\tau))^2}{\lambda_\parallel^2}\right)\mathscr{L}[z_p(t_1)-z_p(\tau)]. \tag{37}$$

The argument of the exponential appearing in Eq. (36) can be evaluated asymptotically, for $z_p(t), z_p(t_1) \gg \lambda_\parallel$, i.e., in the magnetic diffusion regime as

$$8\int_{z_p(t_1)}^{z_p(t)} d\zeta \int_0^\zeta d\zeta_1\, \mathscr{K}(\zeta-\zeta_1) \cong 2\,\frac{z_p(t)-z_p(t_1)}{L_K}, \tag{38}$$

where L_K is the Kolmogorov length:

$$L_K^{-1} = 4\int_0^\infty d\xi\, \mathscr{K}(\xi). \tag{39}$$

All the previous calculations are performed in a given realization of the stochastic parallel velocity $\eta_\parallel(t)$. The solution (36) and (37) still has to be averaged over $\eta_\parallel$. We use the same method as in Ref. [5] for calculating the average over $\eta_\parallel$ of the following function of two variables:

$$F(z-z_1,z_1-\zeta)\equiv\left(1-\frac{(z_1-\zeta)^2}{\lambda_\parallel^2}\right)\mathscr{L}(z_1-\zeta)\exp\left(2\,\frac{z-z_1}{L_K}\right), \tag{40}$$

where $z\equiv z_p(t)$, $z_1\equiv z_p(t_1)$, and $\zeta\equiv z_p(\tau)$. Performing the Fourier transform in the two variables, the average is given by

$$\begin{aligned}\langle F(z-z_1,z_1-\zeta)\rangle_\parallel&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dk\;dq\;F(k,q)\left\langle\exp\left[-ik\int_{t_1}^{t}d\theta\;\eta_\parallel(\theta)-iq\int_{\tau}^{t_1}d\theta\;\eta_\parallel(\theta)\right]\right\rangle_\parallel\\&=\frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty}d\xi_1\;d\xi_2\;F(\xi_1,\xi_2)\int\int_{-\infty}^{\infty}dk\;dq\;\exp[ik\xi_1+iq\xi_2]\\&\quad\times\exp\left[-\frac{1}{2}\,k^2\langle z_p^2(t-t_1)\rangle_\parallel-\frac{1}{2}\,q^2\langle z_p^2(t_1-\tau)\rangle_\parallel-qk\langle z_p(t-t_1)z_p(t_1-\tau)\rangle_\parallel\right].\end{aligned} \tag{41}$$

After integrating over dk, dq, $d\xi_1$, $d\xi_2$, one obtains the following expression for the average of the particle displacement from the magnetic line:

$$\begin{aligned}\langle\Delta r_p^2(t)\rangle_{b\parallel}&=4\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t dt_1\int_0^{t_1}d\tau\left(\frac{\lambda_\parallel^2}{\lambda_\parallel^2+\langle z_p^2(t_1-\tau)\rangle_\parallel}\right)^{3/2}\left(1+4\,\frac{\langle z_p(t-t_1)z_p(t_1-\tau)\rangle_\parallel^2}{L_K^2[\lambda_\parallel^2+\langle z_p^2(t_1-\tau)\rangle_\parallel]}\right)\\&\quad\times\exp\left(2\,\frac{\langle z_p^2(t_1-\tau)\rangle_\parallel\langle z_p^2(t-t_1)\rangle_\parallel-\langle z_p(t-t_1)z_p(t_1-\tau)\rangle_\parallel^2+\lambda_\parallel^2\langle z_p^2(t-t_1)\rangle_\parallel}{L_K^2[\lambda_\parallel^2+\langle z_p^2(t_1-\tau)\rangle_\parallel]}\right).\end{aligned} \tag{42}$$

The decorrelation of the trajectory from the initial field line is produced when the distance $\langle\Delta r_p^2(t)\rangle_{b\parallel}$ is of the order $\lambda_\perp^2$. The model considered here applies to high temperature, weak collisional plasmas. We have neglected the perpendicular collisional velocity in the trajectory equations (8) and (9) but have retained the stochastic ∇b drift. This approximation is justified when $\lambda_{\rm mfp}\beta/\lambda_\parallel>1$. In these conditions, it is natural to consider that the decorrelation is produced during the ballistic regime of the parallel motion and to approximate the parallel mean square displacements appearing in Eq. (42) by $\langle z_p^2(\tau_1)\rangle_\parallel=\frac{1}{2}V_T^2\tau_1^2$, $\langle z_p(\tau_1)z_p(\tau_2)\rangle_\parallel=\frac{1}{2}V_T^2\tau_1\tau_2$. This gives

$$\langle\Delta r_p^2(t)\rangle_{b\parallel}=4\beta^2\,\frac{V_T^4}{\Omega^2\lambda_\parallel^2}\int_0^t dt_1\int_0^{t_1}d\tau\left(1+\frac{V_T^2\tau^2}{2\lambda_\parallel^2}\right)^{-3/2}\left[1+\frac{(V_T^4/\lambda_\parallel^2)t_1^2\tau^2}{L_K^2(1+V_T^2\tau^2/2\lambda_\parallel^2)}\right]\exp\left[\frac{V_T^2t_1^2}{L_K^2(1+V_T^2\tau^2/2\lambda_\parallel^2)}\right]. \tag{43}$$

We thus find a continuous growth of the distance between the particle and the initial magnetic line that is more complicated than in the case of the perpendicular collisional velocity [Eq. (102) in Ref. [5]] but is essentially exponential. The growth rate is related to the exponentiation length of the field lines L_K.

In order to analyze this result it is useful to introduce the following dimensionless quantities: $Y(t)=\langle\Delta r_p^2(t)\rangle_{b\parallel}/\lambda_\perp^2$, $a_1=\lambda_{\rm mfp}/L_K$, $a_2=L_K/\lambda_\parallel$, $c=4\beta^2(\rho_L^2/\lambda_\perp^2)L_K/\lambda_\parallel$, and $\tau=t\nu$. Equation (43) becomes

$$Y(\tau)=c\int_0^{a_1\tau}dx_1\;h(x_1), \tag{44}$$

where

$$\begin{aligned}h(x_1)&=\int_0^{a_2x_1}dx\;\exp\left(\frac{x_1^2}{1+x^2/2}\right)(1+x^2/2)^{-3/2}\\&\quad\times\left(1+2\,\frac{x_1^2x^2}{1+x^2/2}\right).\end{aligned} \tag{45}$$

The integrand in Eq. (45) is a fastly decreasing function of x, which becomes negligible at $x\cong 2$. As a consequence, the function $h(x_1)$ is practically independent of the parameter a_2 [it only affects the behavior of $h(x_1)$ at small x_1 where the values of this function are very small and do not contribute to $Y(t)$]. Thus, the dependence of $Y(t)$ on the parameters enters only through the multiplicative factor c and through the upper limit of the integral in Eq. (44). The numerical calculation of the function $h(x_1)$ showed that it can be approximated by

$$h(x_1)\cong\exp[\epsilon(x_1+x_1^2)],\quad \epsilon=0.767, \tag{46}$$

which gives the following estimate for the dimensionless relative dispersion $Y(t)$:

$$Y(t)=\frac{c}{\epsilon}\frac{1}{1+2a_1\tau}\exp[\epsilon(a_1\tau+a_1^2\tau^2)]. \tag{47}$$

Thus, the curvature stochastic drift determined by the gradients of the fluctuating magnetic field ($a_1\neq 0$) provides an efficient mechanism of decoupling the particles from the field lines producing an exponential departure between the trajectories and the field lines. This result, based on the linearized equation, describes only the first stage in the evolution of the distance particle magnetic line, when $\langle\Delta r_p^2(t)\rangle_{b\parallel}\lesssim\lambda_\perp^2$. Beyond that limit, a diffusion process of the particles is expected. The decorrelation time, defined as the time interval during which $\langle\Delta r_p^2(t)\rangle_{b\parallel}$ grows from 0 to $\lambda_\perp^2$, is determined from the equation $Y(\tau_d)=1$, which has the solution

$$\tau_d=\frac{L_K}{\lambda_{\rm mfp}}\frac{\sqrt{1+4s}-1}{2}, \quad s=\frac{1}{\epsilon}\ln\left(\epsilon_1\frac{\lambda_\parallel^2}{\rho_L^2}\right), \tag{48}$$

where $\epsilon_1=(3/\ln 2)\sqrt{\pi/2}\epsilon=5.42\epsilon$.

The representation of the particle diffusion process as an effective random walk permits us to evaluate the diffusion coefficient as

$$D_{dr}=\frac{\lambda_\perp^2}{2\tau_d}\cong\frac{1}{2}\frac{\lambda_\perp^2V_T}{L_K\sqrt{s}} \quad (\boldsymbol{\eta}_\perp=\mathbf{0}). \tag{49}$$

In the quasilinear limit ($\alpha\equiv\beta\lambda_\parallel/\lambda_\perp\ll 1$), $L_K=\lambda_\perp^2/4D_m$ and the diffusion coefficient (49) can be approximated by

$$D_{dr}=\frac{2\sqrt{\epsilon}D_mV_T}{\ln^{1/2}(\epsilon_1\lambda_\parallel^2/\rho_L^2)} \quad (\boldsymbol{\eta}_\perp=\mathbf{0}), \tag{50}$$

where D_m is the diffusion coefficient of the magnetic lines ($D_m=\sqrt{\pi/2}\beta^2\lambda_\parallel$).

We note that a different result is obtained for a constant velocity motion of the particles along the unperturbed magnetic field [i.e., for $\eta_\parallel(t)=v_\parallel=$const and $V_T=v_\parallel$ in Eqs. (8)–(10)]. From the mathematical point of view, the problem simplifies considerably since the trajectories are determined by a single stochastic function $\mathbf{b}(\mathbf{x})$. The following expression is obtained in this case for the mean square deviation of the trajectories:

$$\langle x_p^2(t)\rangle=2\int_0^{v_\parallel t}d\zeta\,\mathscr{L}^{\rm tr}(\zeta)\left[1+\frac{\rho_L^2}{\lambda_\parallel^2}\left(1-\frac{\zeta^2}{\lambda_\parallel^2}\right)\right](v_\parallel t-\zeta), \tag{51}$$

where $\mathscr{L}^{\rm tr}(\zeta)$ is the Lagrangian correlation of the magnetic field along particle trajectories. Using the method presented in Ref. [5], one can derive an integral equation [similar to Eq. (29)] for $\mathscr{L}^{\rm tr}$:

$$\mathscr{L}^{\rm tr}(z)=\beta^2\exp\left(-\frac{z^2}{2\lambda_\parallel^2}\right)\frac{\lambda_\perp^4}{\{\lambda_\perp^2+2\int_0^z d\zeta\mathscr{L}^{\rm tr}(\zeta)[1+(\rho_L^2/\lambda_\parallel^2)(1-\zeta^2/\lambda_\parallel^2)](z-\zeta)\}^2}. \tag{52}$$

For $\rho_L=0$, Eq. (52) reduces to Eq. (29) and $\mathscr{L}^{\rm tr}(z)$ to $\mathscr{L}(z)$, the Lagrangian correlation of the magnetic field on magnetic lines, which, introduced in Eq. (51), determines the well-known collisionless diffusion coefficient $D_mv_\parallel$. Equation (52) shows that $\rho_L\neq 0$ contributes to the decrease of the Lagrangian correlation and thus of the effective diffusion coefficient. The relative decrease is proportional to $\alpha^2\rho_L^2/\lambda_\parallel^2$, which is rather small for the parameters of tokamak plasmas. This result is in qualitative agreement with Ref. [6] and also with the conclusions deduced in a more general context in Ref. [10]. It applies to collisionless particles, e.g., to the runaway electrons in tokamak plasmas. Thus, the stochastic curvature drifts have an opposite effect on collisionless particles: they reduce the diffusion coefficient while the collisional particle diffusion increase with $\rho_L/\lambda_\parallel$ as seen in Eq. (50).

IV. DIFFUSION DETERMINED BY COLLISIONS AND STOCHASTIC DRIFTS

The perpendicular collisional velocity is now taken into account together with the stochastic curvature drift. The trajectories are thus represented by the complete system (8)–(10) but with a static stochastic field [$\mathbf{b}=\mathbf{b}(\mathbf{x})$ and $\tau_c\to\infty$ in Eq. (14)]. Since $\boldsymbol{\eta}_\perp$ is not correlated with the other two stochastic functions, it determines only an additional free term in Eqs. (31) and (32) for the moments $\langle\Delta x_p^2(t)\rangle_{b\perp}$ and $\langle\Delta y_p^2(t)\rangle_{b\perp}$, respectively, which is of the form $2\int_0^t d\tau\, R_\perp(t-\tau)$. The decorrelation time is calculated following the same steps as in the previous section. With the approximation $\epsilon\cong 1$, the result can be written as a simple, compact expression. Eventually, we obtain the following estimation for particle diffusion coefficient when both the small stochastic drift and the collisional perpendicular velocity are considered:

$$D\cong\frac{2D_mV_T}{\ln^{1/2}\left[\dfrac{1}{(1/16)L_K^2/L_{KP}^2+(1/\epsilon_1)\rho_L^2/\lambda_\parallel^2}\right]} \quad \text{(weak collisions)} \tag{53}$$

where $L_{KP}\equiv\lambda_\perp\sqrt{\chi_\parallel/\chi_\perp}$ is the Kadomtsev-Pogutse characteristic length. In the limit of very small collision frequency $\nu\to 0$ ($\chi_\perp/\chi_\parallel\to 0$ or $L_{KP}\to\infty$) the diffusion coefficient (53) reduces to D_{dr} given by Eq. (50), showing that the particle behavior remains diffusive when the perpendicular collisional diffusivity can be neglected.

The drift induced diffusion coefficient can also be calculated at higher collision frequencies corresponding to the Rechester-Rosenbluth regime (Refs. [3] and [13]). In fact, the perpendicular collisional stochastic velocity could not be neglected in these conditions. However, we study this problem in order to show that the stochastic drifts have a negligible effect at higher collision frequency. The average mean square particle deviation from the magnetic line can be estimated in this case from Eq. (42), using the asymptotic expression $\langle z_p^2(t)\rangle \cong 2\chi_\parallel t$ instead of the ballistic approximation that leads to Eq. (43). The following result is obtained:

$$D_{dr}=\frac{\lambda_\perp^2 \chi_\parallel}{L_K^2 \ln[\lambda_\perp \chi_\parallel/\beta\rho_L V_T L_K]} \quad \text{(strong collisions)}. \tag{54}$$

Comparing Eq. (54) with the collision induced diffusion coefficient [Eq. (98) in Ref. [5]], one finds that D_{dr} is always smaller (as long as $\beta<1/2\sqrt{2}$). Thus, we can conclude that in this high collisional regime the stochastic drifts could not represent the dominant diffusion mechanism. Particle diffusion is mainly determined by the collisional cross field diffusivity and the stochastic drifts contribute only with a (positive) correction to the Rechester-Rosenbluth diffusion coefficient.

V. PARTICLE DIFFUSION IN TIME-DEPENDENT STOCHASTIC MAGNETIC FIELD

We concentrate in this section on the effect of time variation of the stochastic magnetic field on particle diffusion. Therefore, we consider Eqs. (8)–(10) with $\mathbf{b}=\mathbf{b}(\mathbf{x},t)$ [and with finite correlation time τ_c in Eq. (14)], but with $\mathbf{v}_D=\mathbf{0}$ and $\boldsymbol{\eta}_\perp=\mathbf{0}$.

In this case, the mean square displacement $\langle x_p^2(t)\rangle_{b\parallel}$ can be calculated for any t and it is possible to show that its asymptotic behavior is diffusive. Thus we do not need to evaluate the decorrelation time and to perform a random walk estimation of the diffusion coefficient.

The time dependence of the magnetic fluctuations does not affect the instantaneous geometry, i.e., the z dependence of the magnetic lines. It determines, however, a parametrical dependence on time of the Lagrangian correlation of the stochastic magnetic field along magnetic lines. For finite τ_c, the integral equation (29) becomes

$$\mathscr{L}(\zeta,t)=\beta^2 \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right) \times\exp\left(-\frac{|t|}{\tau_c}\right) \frac{\lambda_\perp^4}{[\lambda_\perp^2+2\int_0^\zeta d\zeta_1(\zeta-\zeta_1)\mathscr{L}(\zeta_1,t)]^2}. \tag{55}$$

Introducing the nondimensional quantities $\xi=\zeta/\lambda_\parallel$ and $\mathscr{L}^{nd}(\xi,t)=\mathscr{L}(\xi,t)/[\beta^2\exp(-|t|/\tau_c)]$, Eq. (55) becomes

$$\mathscr{L}^{nd}(\xi,t)=\exp\left(-\frac{\xi^2}{2}\right) \times\frac{1}{[1+2\alpha^2(t)\int_0^\xi d\xi_1(\xi-\xi_1)\mathscr{L}^{nd}(\xi_1,t)]^2}, \tag{56}$$

where $\alpha^2(t)=\beta^2(\lambda_\parallel/\lambda_\perp)^2\exp(-|t|/\tau_c)$. Except for the time dependence in the nonlinearity parameter α, this equation is identical with the equation for the static case (29). Thus, an effect of the time dependence is to gradually decrease the nonlinearity impact as time goes on.

When calculating the mean square deviation of the magnetic lines, one has, of course, to take $t=0$ in Eq. (55), which reduces to the static case, Eq. (29). The time dependence becomes important when particles moving along magnetic lines are considered. Then, a time dependence is introduced in the Lagrangian correlation $\mathscr{L}$ through particle motion $\zeta=z_p(t)$ and the exponential time factor determined by the time variation of the magnetic fluctuations contributes besides particle motion to the Lagrangian correlation of the stochastic field along trajectories.

The mean square particle deviation averaged over the fluctuating magnetic field is

$$\langle x_p^2(t)\rangle_b=\int_0^t\int_0^t dt_1\, dt_2\,\eta_\parallel(t_1)\,\eta_\parallel(t_2) \times\mathscr{L}[z(t_1)-z(t_2),t_1-t_2]. \tag{57}$$

Averaging over the second stochastic parameter $\eta_\parallel$, Eq. (57) becomes

$$\langle x_p^2(t)\rangle_{b\parallel}=\int_0^t\int_0^t dt_1\, dt_2 L_v(t_1-t_2), \tag{58}$$

where

$$L_v(t_1-t_2)=\langle\mathscr{L}[z(t_1)-z(t_2),t_1-t_2]\,\eta_\parallel(t_1)\,\eta_\parallel(t_2)\rangle_\parallel \tag{59}$$

is the Lagrangian correlation of the stochastic perpendicular velocity $\mathbf{v}\equiv\mathbf{b}\eta_\parallel$ determined by particle motion along perturbed field lines. L_v can be calculated by performing a Fourier transform of $\mathscr{L}(z,t)$ in its first argument:

$$L_v(t_1-t_2)=\int_{-\infty}^{\infty}dk\ \mathscr{L}(k,t_1-t_2)\left\langle \eta_\parallel(t_1)\,\eta_\parallel(t_2)\exp\left(-ik\int_{t_2}^{t_1}d\theta\ \eta_\parallel(\theta)\right)\right\rangle_\parallel =\int_{-\infty}^{\infty}dk\ \mathscr{L}(k,t_1-t_2)\,\frac{1}{k^2}\,\frac{\partial^2}{\partial t_1\partial t_2}\left\langle \exp\left(-ik\int_{t_2}^{t_1}d\theta\ \eta_\parallel(\theta)\right)\right\rangle_\parallel. \tag{60}$$

Straightforward calculations consisting in performing the average of the exponential, the time derivatives, the inverse Fourier transform, and eventually the integrations, lead to the following expression for the Lagrangian correlation of the velocity **v**:

$$L_v(t)=\int_{-\infty}^{\infty} d\zeta \ \mathscr{L}(\zeta,t) f(\zeta,t), \tag{61}$$

where

$$f(\zeta,t)=\frac{1}{2}\frac{d}{dt}\left[\frac{d\langle z_p^2(t)\rangle_\parallel}{dt}\frac{1}{\sqrt{2\pi\langle z_p^2(t)\rangle_\parallel}}\times\exp\left(-\frac{\zeta^2}{2\langle z_p^2(t)\rangle_\parallel}\right)\right]. \tag{62}$$

$\langle z_p^2(t)\rangle_\parallel=2\int_0^t d\tau(t-\tau)R(\tau)$ is the mean square deviation in the z direction resulting from Eq. (10) and the correlation function (11). Equation (61) differs from the corresponding equation obtained in the static problem only through the explicit time dependence appearing in the correlation function $\mathscr{L}(\zeta,t)$. We show that this modification produces the transition from the well-known subdiffusive regime of the static case to particle diffusion. To this aim, we note that when $\mathscr{L}=\mathscr{L}(\zeta)$, the time dependence of the correlation $L_v(t)$ coming only from the function $f(\zeta,t)$ has the following shape: for small t it is a positive, decreasing function, then it becomes negative, and at large t it approaches zero asymptotically as $t^{-3/2}$ (Fig. 1). The negative and positive parts of $L_v(t)$ have equal areas so that the diffusion coefficient is zero: $D=\int_0^\infty dt\ L_v(t)=0$. This is the reason for the subdiffusive behavior appearing in the static case. When there is a time variation of the magnetic fluctuations that makes the correlation time τ_c finite, the supplementary time dependence contained in $\mathscr{L}(\zeta,t)$ destroys the ''equilibrium'' of the positive and negative parts of the correlation L_v. Equation (55) shows that the explicit time dependence of $\mathscr{L}(\zeta,t)$ consists in an attenuation that is effective at large t ($t\gtrsim\tau_c$). Thus, the negative part of the correlation is affected more than the positive one and a nonzero, finite diffusion coefficient will result from the time integral of $L_v(t)$ for time-dependent magnetic fluctuations. This image also gives the qualitative behavior of the diffusion coefficient as a function of the correlation time τ_c. Thus, at $\tau_c\to\infty$, the diffusion coefficient is zero and as τ_c decreases it grows due to the fact that the negative part of the correlation L_v is gradually attenuated. A maximum of D is reached at a value of τ_c corresponding to a complete cutoff of the large time, negative part of $L_v(t)$. At smaller values of τ_c, the decay due to time variation of the magnetic fluctuations becomes efficient also on the small time, positive part of L_v so that the diffusion coefficient decreases as $1/\tau_c$ increases.

In the quasilinear limit, at $\alpha(0)\ll 1$, the Lagrangian correlation of **b** on the magnetic lines resulting from Eq. (55) is simply

$$\mathscr{L}(\zeta,t)=\beta^2\ \exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right)\exp\left(-\frac{|t|}{\tau_c}\right) \tag{63}$$

and the Lagrangian correlation of **v** on the trajectories can be calculated easily:

$$L_v(t)=\beta^2\lambda_\parallel\frac{1}{\sqrt{\lambda_\parallel^2+\langle z_p^2(t)\rangle_\parallel}}\left[R(t)-\frac{\varphi^2(t)}{\lambda_\parallel^2+\langle z_p^2(t)\rangle_\parallel}\right]\times\exp\left(-\frac{|t|}{\tau_c}\right). \tag{64}$$

The diffusion coefficient obtained by numerically integrating Eq. (64) is represented in Fig. 2 (by the continuous line) as a function of $\ln(\omega/\nu)$ (where $\omega\equiv 1/\tau_c$). It has indeed the shape described above. As $\tau_c\to 0$ ($\omega\to\infty$), the diffusion coefficient goes to zero. But this is not a correct result since as $\omega\to\infty$, first the time-dependent drift cannot be neglected in Eq. (6) and further, the guiding center approximation is not valid and also the induced electric field becomes important.

Simple analytical expressions for the diffusion coefficient in limited ranges of the parameter ω can be obtained easily from Eq. (64). For the strongly collisional regime characterized by the condition $\gamma\equiv(\lambda_{\mathrm{mfp}}/\lambda_\parallel)^2\ll 1$, the integral of Eq. (64) can be approximated by

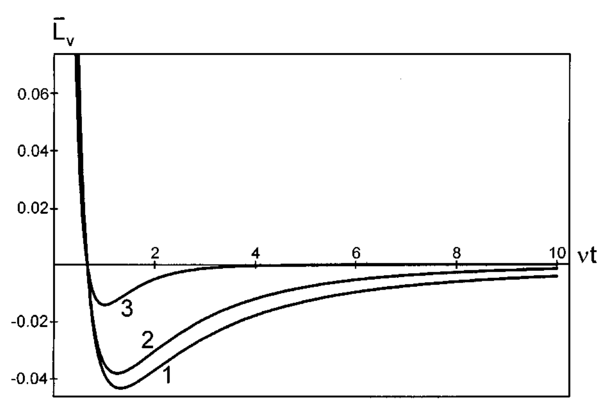

FIG. 1. The Lagrangian correlation $\overline{L_v}=L_v/(\beta^2\chi_\parallel)$ [Eq. (64)] for the following values of ω/ν: 0 (curve 1), 0.1 (curve 2), and 1 (curve 3) [$\gamma\equiv(\lambda_{\mathrm{mfp}}/\lambda_\parallel)^2=20$].

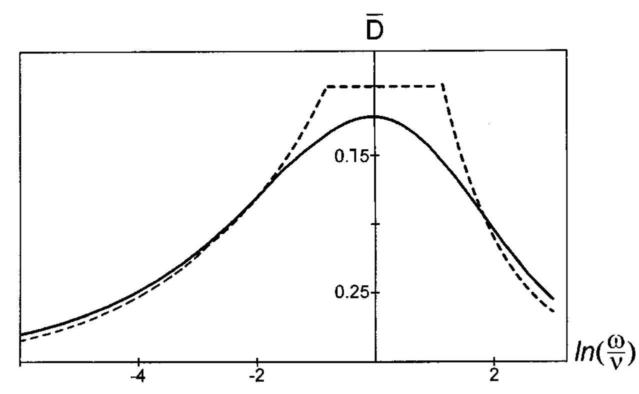

FIG. 2. The normalized particle diffusion coefficient $\overline{D}\equiv D(\omega)/D_{max}$, $D_{\max}=(1/\sqrt{2})\beta^2\lambda_\parallel V_T[1-(2/\gamma)^{1/4}+(1/\gamma)^{1/2}]$ as a function of $\ln(\omega/\nu)$; continuous line: the time integral of Eq. (64); dashed line: the approximation given in Eq. (66) ($\gamma=20$).

$$D(\omega)=\begin{cases}\sqrt{\frac{\pi}{2}}\,\beta^2\lambda_\parallel\sqrt{\chi_\parallel\omega}, & \frac{\omega}{\nu}\ll\gamma\\ \beta^2\chi_\parallel(1-\sqrt{\gamma/2}), & \gamma\ll\frac{\omega}{\nu}\ll 1\,(\gamma\ll 1)\\ \frac{1}{2}\,\beta^2\,\frac{V_T^2}{\omega}, & 1\ll\frac{\omega}{\nu}.\end{cases} \tag{65}$$

Equation (65) shows that an anomalous diffusion is obtained if $\beta^2\chi_\parallel/\chi_\perp>1$, i.e., if $\lambda_{\rm mfp}>\rho_L/\beta$. The two conditions for the mean free path ($\rho_L/\beta<\lambda_{\rm mfp}\ll\lambda_\parallel$) indicate that the magnetic field fluctuations produce a strong anomalous diffusion of the particles in this strongly collisional regime only if $\beta\gg\beta_{\rm lim}\equiv\rho_L/\lambda_\parallel$. The bracket in the second line of Eq. (65) is a correction.

For the weakly collisional regime characterized by the condition $\gamma\gg 1$, the diffusion coefficient can be approximated by

$$D(\omega)=\begin{cases}\sqrt{\frac{\pi}{2}}\,\beta^2\lambda_\parallel\sqrt{\chi_\parallel\omega}, & \frac{\omega}{\nu}\ll 1\\ \frac{1}{\sqrt{2}}\,\beta^2\lambda_\parallel V_T(1-(2/\gamma)^{1/4}+\sqrt{1/\gamma}), & 1\ll\frac{\omega}{\nu}\ll\sqrt{\gamma}\quad(\gamma\gg 1)\\ \frac{1}{2}\,\beta^2\,\frac{V_T^2}{\omega}, & \sqrt{\gamma}\ll\frac{\omega}{\nu}.\end{cases} \tag{66}$$

Again a correction was retained in the second line of this equation. The condition that the maximum value of this diffusion coefficient is larger than $\chi_\perp$ is $\lambda_{\rm mfp}>\rho_L^2/(\beta^2\lambda_\parallel)$. This shows that the diffusion can be anomalous also at levels of the fluctuating magnetic field smaller than the limit $\beta_{\rm lim}$ determined in the previous case. These analytical approximations are represented in Fig. 2 (for $\gamma=20$) together with the numerical integral of the correlation function (64).

The Lagrangian correlation function (64) and the diffusion coefficients (65), (66) are the main results of this section, which concerns $\eta_\perp=0$. Let us end with the following two remarks about the effect of the nonlinearity and the diffusion of purely noncollisional particles, respectively.

The result presented in Fig. 2 is obtained in the limit $\lambda_\perp\to\infty$ (or $\alpha\to 0$) where the Kolmogorov length is infinite. Thus the chaoticity of the stochastic magnetic field does not play an important qualitative role in the diffusion induced by its time variation. Moreover, we have found that nonlinearity parameters $\alpha\lesssim 1$ produce only slight quantitative modifications of the previous, linear results. Numerical integration of Eq. (61) with $\mathscr{L}(\zeta,t)$ given by the numerical solution of Eq. (55) yields a diffusion coefficient very similar to the linear one presented in Fig. 2. The effect of the nonlinearity consists in a small decrease of the value of the maximum together with a slight displacement of it towards larger values of ω.

We also note that particle diffusion in a time-dependent stochastic magnetic field is completely different in the purely collisionless case. When the parallel velocity is constant [$\eta_\parallel(t)=v_\parallel=$const], the Lagrangian correlation of the stochastic perpendicular velocity $\mathbf{v}=\mathbf{b}v_\parallel$ is simply $L_v(t)=v_\parallel^2\mathscr{L}(v_\parallel t,t)$ instead of Eq. (61). As the time dependence of the magnetic fluctuations determines an additional attenuation factor in $\mathscr{L}(\zeta,t)$ [see Eq. (55)], one can immediately deduce that its effect is a continuous decrease of the diffusion coefficient with $\omega\equiv 1/\tau_c$. The effect is as strong as in the case of colliding particles but opposite: the time dependence increases the diffusion of collisional particles for $\omega<\nu$ (see Fig. 2) but strongly reduces the diffusion of collisionless particles. In the quasilinear limit, $\mathscr{L}(\zeta,t)$ is given by Eq. (63) and the following expression results for the collisionless diffusion coefficient:

$$D=D_m v_\parallel\left\{\exp\left(\frac{\omega^2\lambda_\parallel^2}{2v_\parallel^2}\right)\left[1-\mathrm{erf}\left(\frac{\omega\lambda_\parallel}{\sqrt{2}v_\parallel}\right)\right]\right\}, \tag{67}$$

where $\mathrm{erf}(x)$ is the error function. The curly bracket is indeed a decreasing function of ω, always smaller than 1.

VI. COLLISIONAL PARTICLES IN TIME-DEPENDENT MAGNETIC FLUCTUATIONS

In the previous section we have studied particle diffusion in space-time varying stochastic magnetic field neglecting the perpendicular component $\boldsymbol{\eta}_\perp(t)$ of the stochastic velocity determined by collisions. We evaluate here the influence of $\boldsymbol{\eta}_\perp$ on the diffusion coefficient. Thus, the corresponding model consists of Eqs. (3)–(5) with $\mathbf{b}=\mathbf{b}(\mathbf{x},t)$ (τ_c finite), $\mathbf{v}_D=0$, and $\boldsymbol{\eta}_\perp\neq\mathbf{0}$.

This problem cannot be solved using the method of Sec. V since it is not possible to express L_v, the Lagrangian correlation of the velocity $\mathbf{v}=\mathbf{b}\eta_\parallel$, on particle trajectories as a function of $\mathscr{L}(\zeta,t)$, the correlation of the magnetic fluctuations on magnetic lines [Eq. (61)]. In fact, when $\boldsymbol{\eta}_\perp(t)=\mathbf{0}$, the particle paths coincide with the magnetic lines and this is the physical basis of Eq. (61). When $\boldsymbol{\eta}_\perp(t)\neq\mathbf{0}$, one has to determine the Lagrangian correlation of $\mathbf{b}$ on particle paths instead of magnetic lines, which is a much more complicated (unsolved) problem. This can be avoided and the diffusion coefficient can be estimated using the method of Sec. III and of Ref. [5] based on the calculation of the decorrelation time. To this aim, we determine the evolution of the moments $\langle\Delta x_p^2(t)\rangle$, $\langle\Delta y_p^2(t)\rangle$, and $\langle\Delta x_p(t)\Delta y_p(t)\rangle$ where $\Delta\mathbf{x}_p(t)\equiv\mathbf{x}_p(t)-\mathbf{x}_m(z_p(t))$ is the distance between the particle and the magnetic line on which it was initially located. The following equations are obtained in the quasilinear and Markovian approximation:

$$\frac{d}{dt}\langle\Delta x_p^2(t)\rangle_{b\perp}=2\mathscr{M}(t)\langle\Delta x_p^2(t)\rangle_{b\perp}+6\mathscr{M}(t)\langle\Delta y_p^2(t)\rangle_{b\perp}+2\int_0^t d\tau\, R_\perp(t-\tau), \tag{68}$$

$$\frac{d}{dt}\langle\Delta y_p^2(t)\rangle_{b\perp}=6\mathscr{M}(t)\langle\Delta x_p^2(t)\rangle_{b\perp}+2\mathscr{M}(t)\langle\Delta y_p^2(t)\rangle_{b\perp}+2\int_0^t d\tau\, R_\perp(t-\tau), \tag{69}$$

$$\frac{d}{dt}\langle\Delta x_p(t)\Delta y_p(t)\rangle_{b\perp}=-4\mathscr{M}(t)\langle\Delta x_p(t)\Delta y_p(t)\rangle_{b\perp}, \tag{70}$$

where

$$\mathscr{M}(t)-\int_0^t d\theta\,\mathscr{K}[z(t)-z(\theta),t-\theta]\,\eta_\parallel(t)\,\eta_\parallel(\theta). \tag{71}$$

Equations (68)–(70) are identical with Eqs. (75)–(77) obtained in Ref. [5] for the static problem. The only difference consists in the explicit time dependence appearing in the function $\mathscr{K}|z(t)-z\langle\theta\rangle,t-\theta|$. This function determines the Lagrangian correlation of the gradients of **b** and is given by an equation similar with Eq. (30) but with a supplementary time dependence coming from the time variation of **b**:

$$\mathscr{K}(\zeta,t)=\beta^2\exp\left(-\frac{|t|}{\tau_c}\right)\exp\left(-\frac{\zeta^2}{2\lambda_\parallel^2}\right)\frac{\lambda_\perp^4}{[\lambda_\perp^2+2\int_0^\zeta d\zeta_1(\zeta-\zeta_1)\mathscr{L}(\zeta_1,t)]^3}. \tag{72}$$

The solution of Eqs. (68)–(70) with zero initial condition is

$$\langle\Delta x_p^2(t)\rangle_{b\perp}=\langle\Delta y_p^2(t)\rangle_{b\perp}=2\int_0^t dt_1\int_0^{t_1}d\tau\, R_\perp(t_1-\tau)\exp\left[8\int_{t_1}^t d\theta\int_0^\theta d\theta'\;\mathscr{K}[z(\theta)-z(\theta'),\theta-\theta']\,\eta_\parallel(\theta)\,\eta_\parallel(\theta')\right]. \tag{73}$$

This solution still has to be averaged over $\eta_\parallel$. Due to the explicit time dependence of $\mathscr{K}(\zeta,t)$, the averaging method used in Ref. [5] and in Sec. III is not applicable here. As we are interested in determining particle mean square deviation at small time, we can use the cumulant method to the lowest order. The first cumulant of the argument of the exponential in Eq. (73) is, in the quasilinear limit $\alpha\ll 1$ and for $t,t_1\gg 1/\omega$:

$$C_1(\omega;t,t_1)\cong\frac{8}{\lambda_\perp^2}\int_{t_1}^t d\theta\int_0^\theta d\theta'\;L_v(\theta-\theta')\cong\frac{8}{\lambda_\perp^2}D(\omega)(t-t_1), \tag{74}$$

where $D(\omega)$ is the diffusion coefficient in time-dependent stochastic magnetic field for $\boldsymbol{\eta}_\perp=\mathbf{0}$ [Eq. (65) or (66)]. Long but straightforward calculations showed that the time dependence of **b** determines the decay of the second cumulant, which becomes negligible compared to its value for $\omega=0$ when $t,t_1\gg 1/\omega$. It is smaller than the first cumulant except in the limit $\omega\to 0$, where it is dominant and can be approximated by

$$C_2(\omega=0;t,t_1)\cong 32(\pi-2)\beta^4\frac{\lambda_\parallel^2[\langle z_p^2(t)\rangle_\parallel-\langle z_p^2(t_1)\rangle_\parallel]}{\lambda_\perp^4}. \tag{75}$$

Thus, the average over $\eta_\parallel$ of the exponential in Eq. (73) becomes

$$\left\langle\exp\left[8\int_{t_1}^t d\theta\int_0^\theta d\theta'\;\mathscr{K}[z_p(\theta)-z_p(\theta'),\theta-\theta']\,\eta_\parallel(\theta)\,\eta_\parallel(\theta')\right]\right\rangle_\parallel\cong\exp\left[\frac{8a}{\lambda_\perp^2}(t-t_1)\right], \tag{76}$$

where $a=D(\omega)+4(\pi-2)\beta^2\alpha^2\chi_\parallel$. Finally, $\langle\Delta x_p^2(t)\rangle_{b\perp\parallel}$ can be evaluated in the limit of both small and large values of the argument of the exponential in Eq. (76), respectively, the physical meaning of which will be discussed below:

$$\langle\Delta x_p^2(t)\rangle_{b\perp\parallel}\cong\begin{cases}\dfrac{8a\chi_\perp}{\lambda_\perp^2}\,t^2 & \text{small argument}\\[2ex] \dfrac{\lambda_\perp^2\chi_\perp}{4a}\exp\left(\dfrac{8a}{\lambda_\perp^2}\,t\right) & \text{large argument.}\end{cases} \tag{77}$$

The decorrelation time is determined from the equation

$$\langle x_m^2(z_p(t))\rangle_{b\parallel}+\langle\Delta x_p^2(t)\rangle_{b\perp\parallel}=\lambda_\perp^2, \tag{78}$$

where the first term is $\langle x_m^2\rangle_{b\parallel}=2D(\omega)t$. In the quasistaticlimit $\omega\to 0$, the first term is negligible [since $D(0)=0$] and the solution of Eq. (78) is obtained from the second term:

$$t_d \cong \begin{cases} \dfrac{\lambda_\perp^2}{2\sqrt{2a\chi_\perp}} & \text{small argument} \\ \dfrac{\lambda_\perp^2}{8a}\ln\left(\dfrac{4a}{\chi_\perp}\right) & \text{large argument.} \end{cases} \tag{79}$$

The argument of the exponential in Eq. (76) is $\approx 2L_{KP}/L_K$ (where $L_{KP}\equiv\lambda_\perp\sqrt{\chi_\parallel/\chi_\perp}$) at the time given by the first line of Eq. (79) and thus, the validity condition for this result is $L_{KP}/L_K \ll 1$, which corresponds to the Kadomtsev-Pogutse collisional regime (see Ref. [13]). The second line of Eq. (79) was obtained for large values of the argument of the exponential in Eq. (76), which correspond to the condition $L_{KP}/L_K \gg 1$, i.e., to the Rechester-Rosenbluth regime. A random walk estimate of the diffusion coefficient $[D_{\eta_\perp}(\omega) \cong \lambda_\perp^2/2t_d + D(\omega)]$ gives in the limit of small ω:

$$D_{\eta_\perp}(\omega) \cong \begin{cases} \sqrt{2[4(\pi-2)\beta^2\alpha^2\chi_\parallel + D(\omega)]\chi_\perp} + D(\omega), & K\text{-}P \text{ regime} \\ 4\dfrac{4(\pi-2)\beta^2\alpha^2\chi_\parallel + D(\omega)}{\ln\left[4\dfrac{4(\pi-2)\beta^2\alpha^2\chi_\parallel + D(\omega)}{\chi_\perp}\right]} + D(\omega), & R\text{-}R \text{ regime.} \end{cases} \tag{80}$$

$D(\omega)$ is here a small correction to the static values that are recovered as the well-known Kadomtsev-Pogutse or Rechester-Rosenbluth diffusion coefficient, respectively (except for numerical factors of order unity). The upper limit of the frequencies for which the diffusion coefficient (80) is valid can be estimated from the condition that the first term in Eq. (78) is negligible, i.e., from $\langle x_m^2(z_p(t_d))\rangle_{b\parallel} = 2D(\omega)t_d \ll \lambda_\perp^2$. This gives

$$\omega_{\rm lim} \cong \begin{cases} 2\chi_\perp/\lambda_\perp^2, & K\text{-}P \text{ regime} \\ 2\chi_\parallel/L_K^2, & R\text{-}R \text{ regime.} \end{cases} \tag{81}$$

At larger values of ω, the first term in Eq. (78) cannot be neglected: one can show that it becomes dominant compared with the second term and the decorrelation time can be approximated by $t_d \cong \lambda_\perp^2/[2D(\omega)]$, which shows that in this case,

$$D_{\eta_\perp}(\omega) \cong D(\omega). \tag{82}$$

This proves that for high frequency the $\boldsymbol{\eta}_\perp = \mathbf{0}$ results (65) and (66) are valid.

The conclusion of this estimation is that the perpendicular collisional velocity influences the effective particle diffusion only for small ω. For $\omega \gg \omega_{\rm lim}$, $\eta_\perp$ has no significant effect on particle diffusion. We note that $\omega_{\rm lim}/\nu$ is much smaller than γ for $\gamma \ll 1$ and that it is much smaller than 1 for $\gamma \gg 1$, which shows that the small ω range is well before the maximum of $D(\omega)$ [see Eqs. (65) and (66)]. Thus, the effective diffusion coefficient for collisional particles in the time-dependent stochastic magnetic field can be approximated by Eq. (80) for $\omega \ll \omega_{\rm lim}$ and by Eq. (65) or (66) for $\omega \gg \omega_{\rm lim}$. These equations determine four possibilities for the dependence of the diffusion coefficient on ω. At $\omega=0$, $D_{\eta_\perp}$ starts from the Kadomtsev-Pogutse or Rechester-Rosenbluth diffusion coefficient, depending on the value of the parameter L_{KP}/L_K, then it has a slow growth with ω [described by the first or the second line of Eq. (80)]. For $\omega \gg \omega_{\rm lim}$ the growth becomes faster ($\approx\sqrt{\omega}$) and in both cases the diffusion coefficient is $D_{\eta_\perp} = \sqrt{\pi/2}\beta^2\lambda_\parallel\sqrt{\chi_\parallel\omega}$ [i.e., the first line in Eq. (65) or (66)]. Further, at larger ω, the evolution of $D_{\eta_\perp}$ splits into two possibilities depending on the parameter $\gamma \equiv (\lambda_{\rm mfp}/\lambda_\parallel)^2$. They are described by Eqs. (65) and (66). In both cases the diffusion grows with ω up to a maximum and then decays as $1/\omega$ (see Fig. 2) but the position and the amplitude of the maximum are different for the two cases.

These results are in agreement with the heuristic analysis presented in Ref. [9]. A similar maximum of the diffusion coefficient was also obtained (numerically) in Ref. [11] for a single coherent perturbation of the magnetic field.

VII. CONCLUSIONS

We have shown here that, due to the curvature drifts always present along stochastic magnetic lines, the particles decorrelate from the lines, leading to an intrinsic diffusion process. The mechanism of particle-field line decorrelation by the stochastic drifts always contributes, in principle, to particle behavior in a stochastic magnetic field. However, it is the dominant process only when the drift velocity is greater than the root mean squared perpendicular collisional velocity $(\chi_\perp\nu)^{1/2}$. This condition corresponds to high temperature, weakly collisional plasmas having a small cross field collisional diffusivity such that $\chi_\perp < \chi_\parallel(\rho_L\beta/\lambda_\parallel)^2$. In this case the effective diffusion coefficient (50) applies. A clear image of the physical domain corresponding to the validity of the diffusion coefficient (50) induced by the stochastic drifts is presented in Ref. [13] in the context of a systematic analysis of the various diffusion regimes that characterize colliding particles in a stochastic magnetic field.

We have also determined the diffusion coefficient induced by the combined process of drift and collision decorrelation. At high collision frequencies, corresponding to Rechester-Rosenbluth and Kadomtsev-Pogutse regimes, the stochastic drifts do not contribute significantly to the effective diffusion coefficient. However, for weakly collisional plasmas characterized by $L_K \ll \lambda_{\rm mfp}$ we have obtained the result (53) in which both processes are important.

In the second part of this paper, particle diffusion in a time varying stochastic magnetic field is studied. We have shown that the time variation of the magnetic fluctuations has a very strong effect on particle diffusion in all collisional regimes. It consists in a strong increase of the diffusion coefficient. The effect is maximum when the correlation time of the magnetic fluctuations is comparable with the inverse of the collision frequency. When the collisional cross field diffusion is neglected ($\boldsymbol{\eta}_\perp = \mathbf{0}$), the time variation of the magnetic fluctuations determines the transition from the subdiffusive behavior of the particle mean square deviation to a diffusive regime described by Eqs. (65) and (66). As $\omega \equiv 1/\tau_c$ increases, the diffusion coefficient increases up to a value of the order $\beta^2 \chi_\parallel$ or $\beta^2 \lambda_\parallel V_T$, depending on the ratio between the parallel mean free path and the parallel correlation length of the stochastic magnetic field. For faster variation of the magnetic fluctuations, the diffusion coefficient decreases with ω (see Fig. 2). A similar dependence of D on ω is found also for $\chi_\perp \neq 0$ [see Eqs. (80) and (82)]. In the limit $\omega \to 0$, the well-known collision induced diffusion coefficients are recovered (Kadomtsev-Pogutse or Rechester-Rosenbluth). The growth of ω produces an amplification of the diffusion coefficient that becomes practically independent of $\chi_\perp$ and the $\boldsymbol{\eta}_\perp = \mathbf{0}$ diffusion coefficient is found at high ω. The maximum value of the diffusion coefficient can be much larger than the static one and corresponds to correlation times that are in the range of the experimental ones.

We note that two different methods were used in the present paper for deriving the particle diffusion coefficient in time-varying stochastic magnetic field. For $\boldsymbol{\eta}_\perp(t) = \mathbf{0}$ (Sec. V), the problem could be solved exactly (in the frame of Corrsin factorization) while for $\boldsymbol{\eta}_\perp(t) \neq \mathbf{0}$, a random walk estimate based on the calculation of the decorrelation timewas given in Sec. VI.

The results we have obtained by studying particle diffusion induced by pairs of decorrelation mechanisms (stochastic drifts combined with collisions in Sec. IV and time variation of the stochastic magnetic field combined with collisions in Sec. VI, respectively) allow us to draw conclusions on the whole physical process described by the set of equations (8)–(10) containing all the three decorrelation mechanisms. This is possible since we have shown that the effects of the three decorrelation mechanisms can be decoupled in the sense that each is dominant for specified conditions and has a small influence for the range of parameters where another is efficient. For weakly collisional plasmas, the stochastic curvature drifts determine particle behavior and the effective diffusion coefficient is given by Eq. (50). As the collisionality increases, the stochastic drifts lose their importance and the particle diffusion coefficient depends mainly on the collisional cross field diffusivity $\chi_\perp$ [see Eq. (53)]. This happens in static stochastic magnetic fields. For time-dependent stochastic magnetic fields the effect of the perpendicular collisional diffusivity becomes negligible as ω increases and particle diffusion is strongly enhanced as seen in Fig. 2.

The effects described above are generated by the combination of the three stochastic processes: $\mathbf{b}$, $\eta_\parallel$, and $\boldsymbol{\eta}_\perp$. A very important role is played by the first two, which enter multiplicatively into Eqs. (8)–(10). On the other hand, when only one stochastic parameter remains in the trajectory equations, the effect of the stochastic curvature drifts and of the time variation of $\mathbf{b}$ are completely different. This is illustrated by the purely collisionless case [$\eta_\parallel(t) = v_\parallel = \text{const}$, $\boldsymbol{\eta}_\perp(t) = \mathbf{0}$] for which we have obtained from Eq. (52) a weak decrease of the basic collisionless diffusion coefficient ($D_m v_\parallel$) induced by the stochastic curvature drifts, in agreement with Refs. [10] and [6]. As for the effect of the time variation of the stochastic magnetic field, it consists in a continuous decay of the diffusion coefficient as ω increases [Eq. (67)].

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